STATS Chapter 8: Confidence Intervals

Review of key concepts:

The Empirical Rule and the Normal Curve (Section 2-1)

___________% of observations will be within 1 Standard Deviation of the mean.

___________% of observations will be within 2 Standard Deviations of the mean.

___________% of observations will be within 3 Standard Deviations of the mean.

Rules for inference (Section 4-3)

Were the individuals randomly selected? If yes, then you can make inferences about ___________

Were the treatments randomly assigned? If yes, then you can make inferences about ___________

Section 8-1: Confidence Intervals: The Basics

If we want to estimate the value of some parameter from a population, we can take a sample and calculate a statistic from that sample. The sample statistic is an estimate of the population parameter. But we know from the previous chapters and from common sense that selecting different samples will yield different statistics and therefore different estimates for the population parameter. It would therefore make more sense to, instead of giving just one estimate of the population parameter to give an entire interval of reasonable values based on the sample data.

Example:
Suppose I take an SRS of 35 seniors at NWHS and ask them how many colleges they applied to. Suppose the mean, \( \bar{x} \), of this SRS was 5 colleges. It is very unlikely that if I were to sample the entire senior class that the average number of colleges applied to would be exactly 5. Therefore, I am going to say that I am pretty confident, say 95% confident, that the average number of colleges that seniors applied to, while not exactly 5, is somewhere between 3 and 7. How I got the interval from 3 to 7 and how I got the “95% confidence” we’ll discuss later, but at least the concept of what we call a confidence interval is demonstrated in this example.

A confidence interval for a population parameter is an interval of plausible values for that parameter. It is constructed in such a way so that, with a chosen degree of confidence, the value of the parameter will be captured inside the interval.

In the above example, the confidence interval was 3 to 7.

The chosen degree of confidence is called the confidence level. The confidence level gives information about how much “confidence” we have in the method used to construct the interval. In other words, a confidence level gives the success rate of the method used to construct the interval.

In the above example, the confidence level was 95%
Constructing a confidence interval for a population mean (pop. $\sigma$ is known):

**Step 1:** Have you met the conditions for using a confidence interval?

- **Randomly selected sample** – Look for the term SRS. Without random selection, we lose the ability to make inferences about the population.

- **Normal Distribution** – We will be using Normal Curves to find probabilities, so we need the distribution to be Normal.
  
  For Means: either the population has to be normal, or the sample size $n$ must be at least 30 CLT.
  
  For Proportions: $np \geq 10$ and $n(1-p) \geq 10$

- **Independent Observations** – In order to use our standard deviation formula, we need the sample size to be less than 10% of the population size, so $10n > N$. This is especially true when sampling without replacement.

**Step 2:** Decide what confidence level you want. Usually the confidence level is 90%, 95% or 99%. For our example, let’s construct a 95% confidence interval.

**Step 3:** Find the sample mean, $\bar{x}$, used to estimate the population mean, $\mu$, using an SRS. $\mu = \mu_{\bar{x}}$

**Step 4:** Calculate the Standard Deviation of the mean: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ or $\sigma_{\bar{x}} = \frac{s_{\bar{x}}}{\sqrt{n}}$

**Step 5:** The distribution of these sample means is approximately normal with approximately 95% of all $\bar{x}$ values within 2 standard deviations of the mean of that distribution, (the actual z value is 1.96). So approximately 95% of all $\bar{x}$ values will be between $-1.96\sigma_{\bar{x}}$ and $1.96\sigma_{\bar{x}}$.

Since $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, we can expect that approximately 95% of all $\bar{x}$ values will be between $-1.96\frac{\sigma}{\sqrt{n}}$ and $1.96\frac{\sigma}{\sqrt{n}}$

**Step 6:** State the Confidence Interval: $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

**Step 7:** State the AP Language:

We are 95% confident that the interval $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}$ to $\bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$ captures the true population parameter $\mu$.

OR If we take many samples, the true population parameter, $\mu$, will be captured by approximately 95% of the intervals.
A Note about standard deviation when working with sample means:

If the standard deviation of the population is known, use $\sigma$.

If the standard deviation of the population is not known, it will be calculated from the data. This is known as the **standard error**. Use $S_x$.

Note: Sometimes additional sources of error exist. This could make the true interval wider. Examples: non-response bias, lying, volunteer response bias.

### What does it mean to capture the population parameter?

Look at the intervals to the right $\rightarrow$

$\mu$ is the population parameter.

Some intervals “capture” $\mu$ and some do not capture $\mu$.

At the 95% confidence level, we expect 95 out of 100 intervals to capture $\mu$ and 5 to not capture $\mu$.

Remember: the confidence interval describes the mean or proportion of a population, NOT individuals.

### Common Z-Values

<table>
<thead>
<tr>
<th>Confidence</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>1.28</td>
</tr>
<tr>
<td>90%</td>
<td>1.64</td>
</tr>
<tr>
<td>92.5%</td>
<td>1.78</td>
</tr>
<tr>
<td>95%</td>
<td>1.96</td>
</tr>
<tr>
<td>97.5%</td>
<td>2.24</td>
</tr>
<tr>
<td>99%</td>
<td>2.58</td>
</tr>
<tr>
<td>* (most common) *</td>
<td></td>
</tr>
</tbody>
</table>
Examples:

I survey 20 students in detention to find the proportion of students with internet access at home. 65% of these students have internet access. Is this a good predictor of the population proportion?

Find the critical $z$-value for the following confidence levels

$$75\% = \quad 85\% = \quad 98\% =$$

The average price of 40 recently purchased Darien homes (taken via an SRS) was $730,000. Assume that the true standard deviation of Darien home prices (of homes recently purchased) is $\sigma = 60,000$. Construct a 95% confidence interval for the true average price of all recently purchased homes in Darien.

AP Wording: we are 95% confident that the true mean home price is captured in the interval from…

Quality control inspectors want to estimate the mean lifetime $\mu$ of the AA batteries produced in an hour at a factory. They select a random sample of 30 batteries during each hour of production and then drain them under conditions that mimic normal use. Here are the lifetimes (in hours) of the batteries: (also on page 470)

16.91  18.83  17.58  15.84  17.42  17.65  16.63  16.84  15.63  16.37  15.80  15.93
15.98  16.52  17.04  17.07  15.73  16.74

Construct a 90% confidence interval using $S_x$ for the standard deviation.

AP Wording: if followed the same procedure many times, we would expect the true mean to be captured by…

HW A: read 468 – 476; questions 5, 7, 9, 11, 13
Constructing a confidence interval for a population proportion:

**Step 1:** Have you met the conditions for using a confidence interval?

- **Randomly selected sample** – Look for the term SRS. Without random selection, we lose the ability to make inferences about the population.

- **Normal Distribution** – We will be using Normal Curves to find probabilities, so we need the distribution to be Normal.
  
  For Means: either the population has to be normal, or the sample size n must be at least 30 CLT.
  
  For Proportions: np ≥ 10 and n(1-p) ≥ 10

- **Independent Observations** – In order to use our standard deviation formula, we need the sample size to be less than 10% of the population size, so 10n > N. This is especially true when sampling without replacement.

**Step 2:** Decide what confidence level you want. Usually the confidence level is 90%, 95% or 99%. For our example, let’s construct a 95% confidence interval.

**Step 3:** Find the sample proportion, \( \hat{p} \), used to estimate the population proportion, p, using an SRS. \( \mu_\hat{p} = p \)

**Step 4:** Calculate the Standard Deviation of the sample proportion:

\[
\sigma_\hat{p} = \sqrt{\frac{p(1-p)}{n}}
\]

**Step 5:** The distribution of these sample proportions is approximately normal with approximately 95% of all \( \hat{p} \) values within 2 standard deviations of the mean of that distribution (the actual z value is 1.96). So approximately 95% of all \( \hat{p} \) values will be between \(-1.96\sigma_\hat{p}\) and \(1.96\sigma_\hat{p}\).

Since \(\sigma_\hat{p} = \sqrt{\frac{p(1-p)}{n}}\), we can expect that approximately 95% of all \( \bar{x} \) values will be between \(-1.96\sigma_\hat{p}\) and \(1.96\sigma_\hat{p}\)

**Step 6:** State the Confidence Interval: \( \hat{p} \pm 1.96 \sqrt{\frac{p(1-p)}{n}} \)

**Step 7:** State the AP Language:

We are 95% confident that the interval \( \hat{p} - 1.96 \sqrt{\frac{p(1-p)}{n}} \) to \( \hat{p} + 1.96 \sqrt{\frac{p(1-p)}{n}} \) captures the true population parameter p. OR If we take many samples, the true population parameter, p, will be captured by approximately 95% of the intervals.
Day 2

Opener – read and answer questions on weather predictions.

Examples:

**Problem**: Mr. Vignolli’s class took a SRS of beads from the container and got 107 red beads and 144 white beads.

(a) Calculate and interpret a 90% confidence interval for $p$.

(b) Mr. Vignolli claims that exactly half of the beads in the container are red. Use your result from (a) to comment on this claim.

*Kissing the right way?*

According to an article in the *San Gabriel Valley Tribune* (February 13, 2003), “Most people are kissing the ‘right way.’” That is, according to the study, the majority of couples tilt their heads to the right when kissing. In the study, a researcher observed a random sample of 124 couples kissing in various public places and found that 83/124 (66.9%) of the couples tilted to the right. Construct and interpret a 95% confidence interval for the proportion of all couples who tilt their heads to the right when kissing.

The 2007 Youth Risk Survey questioned a random sample of 14,041 students in grades 9-12. Of these, 2808 said they had smoked at least one day in the past month. Construct a 99% confidence interval of the proportion of teen smokers.

HW B: read 476 – 481 and 484 - 487; questions 17, 19-24, 27, 31, 33
Day 3

III. Inferential Statistics

On the AP Formula Page:

Standardized test statistic: \( \frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}} \)

Confidence interval: statistic \( \pm \) (critical value) \( \cdot \) (standard deviation of statistic)

How Confidence Intervals Behave:

1) When designing confidence intervals, a researcher wants to accomplish 2 things:

   a) Have a high confidence level

      This is relatively subjective; the researcher decides what confidence level they wish. This confidence level, however, can drive other factors about the experiment.

   b) Have a narrow confidence interval

      In order to attain a narrow confidence interval the margin of error \( (z \text{ critical value} \cdot \frac{\sigma}{\sqrt{n}}) \) must be reduced. The way to reduce this value is by reducing the \( z \) critical value. But reducing the \( z \) critical value would also mean lowering your level of confidence. So there is a tradeoff between confidence level and margin of error (assuming the sample size stays constant). Another option for reducing the margin of error is increasing the sample size. This is a good option because it allows you to reduce the margin of error while maintaining your desired level of confidence. The drawback is that collecting a larger sample size might be too costly or time consuming.

Example:
The confidence interval at the 95% level is 4 to 6.

What happens if the confidence level is changed to 80%

How would we change the confidence level so that the confidence interval is smaller?

What else could we do to shrink the confidence interval?
Example:
Let’s refer back to our last example of recently purchased Darien homes. The average purchase price from an SRS of size 40 was $730,000. The standard deviation is still $60,000. We found in the last example that a 95% confidence interval for the price of a recently sold home was ($711,406, $748,594).

a) Construct a 90% confidence interval for the same data. What do you notice about this interval compared to the 95% confidence interval?

b) Construct a 99% confidence interval for the same data. What do you notice about this interval compared to the 95% confidence interval?

c) Lets say a real estate agent comes to you and asks, “how confident are you, based on your data, that recent home sales in Darien are somewhere between $720,000 and $740,000?

Choosing the Sample Size when sampling to estimate a population mean:
Sometimes, researchers need to both limit the margin of error to a fixed amount and still maintain a certain confidence level. In such cases, the only thing that can vary is the sample size. Choosing the appropriate sample size under confidence level and margin of error constraints can be easily found using algebraic manipulation of the margin of error piece of a confidence interval:

\[
\text{Margin of error } = m = (z \text{ critical value}) \cdot \frac{\sigma}{\sqrt{n}}
\]

\[
\sqrt{n} \cdot m = (z \text{ critical value}) \cdot \sigma
\]

\[
\sqrt{n} = \frac{(z \text{critical value}) \cdot \sigma}{m}
\]

\[
n = \left( \frac{(z \text{critical value}) \cdot \sigma}{m} \right)^2
\]
Choosing the Sample Size when sampling to estimate a population mean:

When we designed confidence intervals for population means we often times researched ahead of time how large of a sample we would need in order to get a certain margin of error with certain confidence. This notion also holds true when designing confidence intervals for proportions. The following are the algebraic steps in deriving the formula:

The margin of error for a confidence interval for p is

\[ m = z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}. \]

Since we don’t know p-hat until we actually conduct the study we have a problem. We can then do one of two things:

a) Use an estimated p-hat based on previous studies

b) Use a conservative p-hat = 0.5. Using a p-hat of 0.5 gives the largest possible margin of error for any given z or n. We will use this option most of the time.

Square both sides:

\[ m^2 = (z^*)^2 \left( \frac{0.5(1 - 0.5)}{n} \right) \]

Divide both sides by \((z^*)^2 \cdot 0.5\): \[ \frac{m^2}{(z^*)^2 \cdot 0.25} = \frac{1}{n} \]

Cross multiply and divide:

\[ n = \frac{0.25 \cdot (z^*)^2}{m^2} \quad \text{NOTE: AP STAT Program does this!} \]

Examples:

Again refer to the Darien home purchase example from before. Let’s say you are a researcher that was hired by a brokerage company to study the average price of recently sold homes in Darien. The brokerage company wants to be 95% confident in your results. Furthermore, the company wants the margin of error in your interval to be no more than $20,000. How large of a sample must you collect to get this information?

Before conducting nationwide polling prior to a national election, most good companies will decide ahead of time how many people to include in the poll based on a particular margin of error and confidence interval. Lets say that Gallop wants to conduct a poll to see what proportion of the voters would vote for Bush if George Bush were running for president today against John Kerry? Gallop wants to be 95% confident with a margin of error no more than 3 percentage points. How many people must they randomly sample in order to achieve this?

A Gallop poll asking 1,200 people if they have a tattoo reports that the 38% percent have a tattoo with a margin of error of ±.02453 Find the confidence level.

HW C: read 487 – 495; questions 35, 37, 41, 43, 47
Previously, when constructing confidence intervals, we made the assumption that we knew the true population standard deviation, $\sigma$. This, of course, is a silly assumption. When we do not know the population standard deviation, $\sigma$, we must estimate it from our sample using the sample standard deviation, $s$. However when we do so, the test statistic, $z$, that we previously used, changes. The new test statistic is now called the $t$-statistic, and has a new distribution (density curve) associated with it.

If you draw an SRS from a population that has a normal distribution with mean, $\mu$, and standard deviation, $\sigma$, the $t$-statistic:

$$ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} $$

has the $t$ distribution with $n - 1$ degrees of freedom.

The new distribution is not exactly like the standard normal curve, but is very close:

- It is still centered at zero
- It is bell shaped
- Its spread is slightly greater than the standard normal distribution
- It has more probability in the tails
- It is dependent on the sample size used to find $s$. This dependence on sample size is taken care of via “degrees of freedom”.
Constructing a confidence interval for a population mean (pop. \( \sigma \) is not known):

Step 1: Have you met the conditions for using a confidence interval?

**Randomly selected sample** – Look for the term SRS. Without random selection, we lose the ability to make inferences about the population.

**Normal Distribution** – We will be using t-distribution (Normal Curve with more variation) to find probabilities, so we need the distribution to be approximately Normal.

For Means: either the population has to be normal, or the sample size \( n \) must be at least 30 CLT.

For Proportions: \( np \geq 10 \) and \( n(1-p) \geq 10 \)

**Independent Observations** – In order to use our standard deviation formula, we need the sample size to be less than 10% of the population size, so \( 10n > N \). This is especially true when sampling without replacement.

Step 2: Decide what confidence level you want. Usually the confidence level is 90%, 95% or 99%. For our example, let’s construct a 95% confidence interval.

Step 3: Find the sample mean, \( \bar{x} \), used to estimate the population mean, \( \mu \), using an SRS. \( \mu_{\bar{x}} = \mu \)

Step 4: Calculate the Standard Deviation of the mean: 
\[
\sigma_{\bar{x}} = \frac{s_{\bar{x}}}{\sqrt{n}}
\]

Step 5: The distribution of these sample means follows a t-distribution with approximately 95% of all \( \bar{x} \) values within 2 standard deviations of the mean of that distribution, (the actual critical t value varies based on \( n \)). Use the calculator or t-distribution table to find the critical t-values based on degrees of freedom: \( df = n - 1 \)

Step 6: State the Confidence Interval: 
\[
\bar{x} \pm t \cdot \frac{s_{\bar{x}}}{\sqrt{n}}
\]

Step 7: State the AP Language:

We are 95% confident that the interval \( \bar{x} - t \cdot \frac{s_{\bar{x}}}{\sqrt{n}} \) to \( \bar{x} + t \cdot \frac{s_{\bar{x}}}{\sqrt{n}} \) captures the true population parameter \( \mu \). OR If we take many samples, the true population parameter, \( \mu \), will be captured by approximately 95% of the intervals.
Note: $\frac{s}{\sqrt{n}}$ is called the standard error of the estimate (SE estimate for short).

State the T-Values

Confidence Level = 90%, degrees of freedom = 3

Confidence Level = 60%, degrees of freedom = 2

Confidence Level = 98%, degrees of freedom = 5

Examples:

*How much homework?*

The principal at a large high school claims that students spend at least 10 hours per week doing homework, on average. To investigate this claim, an AP Statistics class selected a random sample of 250 students from their school and asked them how long they spent doing homework during the last week. The sample mean was 10.2 hours and the sample standard deviation was 4.2 hours.
George is attempting to find out how many text messages the average NWHS student sends in a month. He takes an SRS of 15 NWHS students and gets the following numbers:

<p>| | | | | |</p>
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<tbody>
<tr>
<td>107</td>
<td>116</td>
<td>153</td>
<td>56</td>
<td>44</td>
</tr>
<tr>
<td>187</td>
<td>36</td>
<td>145</td>
<td>107</td>
<td>29</td>
</tr>
<tr>
<td>109</td>
<td>87</td>
<td>138</td>
<td>121</td>
<td>91</td>
</tr>
</tbody>
</table>

So we will construct a 95% confidence interval for the average number of text messages that a DHS sends in a month.

Note: Since our sample size is less than 30 we need to make sure we do not have outliers and that our sample comes from a normal population. Create a box-plot, a histogram, and a normal probability plot.

A manufacturer of high-resolution video terminals must control the tension on the mesh of fine wires that lies behind the surface of the viewing screen. Too much tension will tear the mesh, and too little will allow wrinkles. The tension is measured by an electrical device with output readings in millivolts (mV). Some variation is inherent in the production process. Here are the tension readings from a random sample of 20 screens from a single day's production:

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>269.5</td>
<td>297.0</td>
<td>269.6</td>
<td>283.3</td>
<td>304.8</td>
<td>280.4</td>
<td>233.5</td>
</tr>
<tr>
<td>264.7</td>
<td>307.7</td>
<td>310.0</td>
<td>343.3</td>
<td>328.1</td>
<td>342.6</td>
<td>338.8</td>
</tr>
<tr>
<td></td>
<td>340.1</td>
<td>374.6</td>
<td>336.1</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Construct and interpret a 90% confidence interval for the mean tension $\mu$ of all the screens produced on this day.

HW D: read 499 – 507; questions 49-52, 55, 57, 59, 63
Robust Procedures: A confidence interval or hypothesis test is called robust if the confidence level or p-value does not change very much when the assumptions of the procedure are violated. This means that the conditions to use the t-test are not met (such as a non-normal population), but the results are still fairly accurate.

- The t procedure is quite robust against non-normality of the population
- The t procedure is NOT robust when there are outliers
- Results improve as sample size, n, increases

General Rules based on sample size:

- \( n < 15 \) only use t-test if population is normal and there are no outliers
- \( 15 < n < 30 \) use t-test unless there are outliers or the population has a strong skew
- \( n > 30 \) use t-test even when skewed, beware of outliers

Examples

Problem: Determine whether we can safely use a one-sample t interval to estimate the population mean in each of the following settings.

(a) Figure 8.15(a) is a histogram of the percent of each state's residents who are at least 65 years of age.

(b) Figure 8.15(b) is a stemplot of the force required to pull apart 20 pieces of Douglas fir.

Problem: Determine whether we can safely use a one-sample t interval to estimate the population mean in each of the following settings.

(a) To estimate the average GPA of students at your school, you randomly select 50 students from classes you take. Here is a histogram of their GPAs:

(b) The dotplot below shows the amount of time it took (in minutes) to order and receive a regular coffee in five visits to a local coffee shop.

(c) The boxplot below shows the SAT Math scores for a random sample of 20 students at your high school.
Examples

Researchers would like to estimate the mean cholesterol level $\mu$ of a particular variety of monkey that is often used in laboratory experiments. They would like their estimate to be within 1 milligram per deciliter (mg/dl) of the true value of $\mu$ at a 95% confidence level. A previous study involving this variety of monkey suggests that the standard deviation of cholesterol level is about 5 mg/dl.

CHECK YOUR UNDERSTANDING

To assess the accuracy of a laboratory scale, a standard weight known to weigh 10 grams is weighed repeatedly. The scale readings are Normally distributed with unknown mean (this mean is 10 grams if the scale has no bias). In previous studies, the standard deviation of the scale readings has been about 0.0002 gram. How many measurements must be averaged to get a margin of error of 0.0001 with 98% confidence? Show your work.

As part of their final project in AP Statistics, Christina and Rachel randomly selected 18 rolls of a generic brand of toilet paper to measure how well this brand could absorb water. To do this, they poured 1/4 cup of water onto a hard surface and counted how many squares it took to completely absorb the water. Here are the results from their 18 rolls:

<table>
<thead>
<tr>
<th>29</th>
<th>20</th>
<th>25</th>
<th>29</th>
<th>21</th>
<th>24</th>
<th>27</th>
<th>26</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>24</td>
<td>27</td>
<td>28</td>
<td>21</td>
<td>25</td>
<td>26</td>
<td>22</td>
<td>23</td>
</tr>
</tbody>
</table>

Construct and interpret a 99% confidence interval for $\mu$ = the mean number of squares of generic toilet paper needed to absorb 1/4 cup of water.
Environmentalists, government officials, and car makers study the exhaust emissions produced by motor vehicles. Researchers collected data on the nitrogen oxides (NOX) levels in grams/mile for a random sample of 40 light-duty engines of the same type. The mean NOX reading was 1.2675 and the standard deviation was 0.3332.

a) Construct and interpret a 95% confidence interval for the mean amount of NOX emitted by light-duty engines of this type.

b) The EPA sets a limit of 1 gram/mile for NOX emissions. Are you convinced that this type of engine has a mean NOX level of 1 or less?

c) The original sample included 41 engines, but one was removed due to an extreme NOX level (2.94) due to a mechanical defect. The printout below shows the data with the engine included.

<table>
<thead>
<tr>
<th>NoX</th>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>NoX</td>
<td>41</td>
<td>1.3083</td>
<td>0.0556</td>
<td>0.4201</td>
<td>0.5700</td>
<td>1.1000</td>
<td>1.3100</td>
<td>1.4700</td>
<td>2.9400</td>
</tr>
</tbody>
</table>

How would including the value affect the confidence interval? Find the confidence interval with this engine included. Does this change your answer to part b?